

# RESEARCH MEMORANDUM

DESIGN DATA FOR GRAPHICAL CONSTRUCTION

OF TWO-DIMENSIONAL SHARP-EDGE-THROAT

SUPERSONIC NOZZLES

By Harold Shames and Ferris L. Seashore

Lewis Flight Propulsion Laboratory Cleveland, Ohio

# NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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In table 1 page 15, the  $\beta$  (column 3) value for  $\psi^-$  of .46 should be 19.99 instead of 19.90.

The  $\frac{y}{At/2}$  values (column 5) for  $\psi^-$  43.00, 45.00 and 47.00

should be 7.73, 5.77, and 1.80 instead of 3.87, 2.88, and .90.

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OF TWO-DIMENSIONAL SHARP-EDGE-THROAT

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#### SUMMARY

Design data are presented for the graphical construction of two-dimensional sharp-edge-throat supersonic nozzles of minimum length for test-section Mach numbers from 1.20 to 10.00. The method of characteristics used in the design is briefly reviewed.

#### INTRODUCTION

A general discussion of the method of characteristics as applied to supersonic-nozzle design is given in references 1 to 3. The application of the method of characteristics to the design of minimum-length sharp-edge-throat nozzles is described in reference 3.

By means of charts and tables presented herein for designing such nozzles using an expansion "kernel," nozzle-wall contours for wind-tunnel test-section Mach numbers from 1.20 to 10.00 may be obtained with a minimum of graphical construction. The principles of the method of characteristics used in the design are reviewed. The nomenclature of reference 1 was found to be more convenient than the speed-index or pressure-number systems of references 2 and 3, and is therefore used in this report.

#### SYMBOLS

The following symbols are used in this report:

- Area of nozzle bearing uniform flow at Mr (equal to height for nozzle of unit width)
- At area of nozzle at throat (equal to height for nozzle of unit width)

- L length of nozzle from throat to test section
- lk length of kernel
- M Mach number
- Mr final Mach number
- x abscissa of point of intersection of  $\psi_{\hat{I}}^+$  characteristic with  $\psi^-$  characteristic
- y ordinate of point of intersection of  $\psi_{\mathbf{f}}^-$  characteristic with  $\psi^-$  characteristic
- angle of wall to nozzle axis
- $\beta$  Mach angle,  $\left(\sin^{-1}\frac{1}{M}\right)$
- $\beta_f$  final Mach angle
- 7 ratio of specific heat at constant pressure to specific heat at constant volume
- $\theta$  angle of inclination of streamline to nozzle axis
- $\lambda^+$  angle that  $\psi^+$  characteristic makes with x axis  $(\beta \theta)$   $(\lambda^+$  is positive number when drawn below horizontal)
- $\lambda^-$  angle that  $\psi^-$  characteristic makes with x axis  $(\beta + \theta)$   $(\lambda^-$  is positive number when drawn above horizontal)
- φ angle of corner in wall at nozzle throat
- ψ equivalent Prandtl-Meyer turning angle
- $\psi^+$  characteristics (Mach waves) originating at upper nozzle wall
- Ψ characteristics (Mach waves) originating at lower nozzle wall
- $\psi_{_{\mathcal{P}}}$  value of  $\psi$  at nozzle exit
- $\Psi_{\mathbf{f}}^{+}$  downstream characteristic bounding expansion wave originating at upper nozzle wall
- downstream characteristic bounding expansion wave originating at lower nozzle wall

#### METHOD OF NOZZLE DESIGN

System of Characteristics in Sharp-Edge-Throat Kozzles

A two-dimensional nozzle with a sharp-edge throat is shown in figure 1. The increase in flow Mach number with displacement downstream of the throat is obtained from the system of expansion waves generated at the angular turn of the wall at the nozzle throat (fig. 2(a)). The expansion waves, as shown in figure 2(a). turn the flow toward the adjacent nozzle wall downstream of the corner with a consequent increase in stream-tube cross-sectional area and Mach number. The system of expansion waves from each corner is identical with that developed in an infinite uniform sonic flow constrained to flow around a sharp corner in a single two-dimensional wall. The solution for this case is discussed in reference 4. The expansion waves are propagated into the flow along straight lines radiating from the corner in the case for the flow along only one wall in an infinite flow. Along any given radial line, the flow direction, the Mach number, and the physical state of the gas is the same for all points on that line (fig. 2(a)). Each of these radial lines can be assigned a number in degrees or radians that corresponds to the angular deviation of the flow crossing the line from the direction of the undisturbed sonic flow. A line so numbered is called a characteristic. The angular deviation of the flow between two characteristics is equal to the difference of the numbers assigned to these characteristics. At each characteristic, the flow makes the Mach angle  $\beta = \sin^{-1} 1/M$  with the characteristic. The characteristics are therefore coincident with the Mach lines in the flow.

Two separate walls in the flow (fig. 2(a)) result in two separate systems of intersecting expansion waves originating at the respective wall corners. If the characteristics from the upper and lower walls are designated  $\psi^+$  and  $\psi^-$ , respectively, every point in the flow traversed by both expansion waves is crossed by a characteristic from the upper and lower walls. Because of the simultaneous influence on the flow of the expansion waves from the corner on the upper and lower walls in the zone common to both sets of waves, the characteristics are curved to maintain the Mach angle with the flow (zone I, fig. 2(b)). The characteristics are straight in zones occupied by only one set of expansion waves (zones II and III, fig. 2(b)).

By means of the characteristics in zones II and III, the graphical construction of the nozzle-wall contour required to

give wave-free flow in the test section can be made. Tables I and II provide the information for obtaining the characteristics in zones II and III without involved plotting or computation. The construction of the wave pattern from which the information in tables I and II was obtained is described in the following section.

# Development of Kernel

From references 2 and 4, the value of the flow Mach number at a point in the flow, crossed by characteristics having values of  $\psi^+$  and  $\psi^-$ , respectively, is given by

$$\Psi = \Psi^{+} + \Psi^{-} = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \frac{\sqrt{M^{2}-1}}{\sqrt{\frac{\gamma+1}{\gamma-1}}} - \tan^{-1} \sqrt{M^{2}-1}$$
 (1)

The flow direction with respect to the nozzle axis is

$$\theta = \psi^+ - \psi^- \tag{2}$$

For an isentropic flow of known uniform total pressure and temperature, the flow at any point is completely specified by the local values of the intersecting pair of characteristics.

A wave pattern for a pair of opposite corners at the nozzle throat is established by dividing the wave emitted by each corner into a convenient number of characteristics, and by determining the resulting wave pattern due to the interaction of both sets of waves by means of the foregoing principles; that is, the local Mach number is given by equation (1), the flow direction is given by equation (2), and the local Mach angle is determined from the relation  $\theta = \sin^{-1} 1/M$ .

The resulting system of characteristics in the zone of the flow traversed by waves from both corners (zone I, fig. 2(b)) is shown schematically in figure 2(c). Such a pattern is called a kernel. In order to obtain the tables giving the pertinent design parameters for sharp-edge nozzles ranging in test-section Mach number from 1.20 to 10.00, a kernel was graphically developed for two opposing corners of equal angle (51.16°) corresponding to M = 10.00 at the test section with the following increments in  $\psi^+$  and  $\psi^-$ :

988

In the range of low values of  $\psi^+$  and  $\psi^-$ , where the construction is sensitive to small changes in these values, small increments in  $\psi^+$  and  $\psi^-$  were used, as indicated in the preceding table.

Because the corners at the nozzle throat were chosen equal, the resultant wave pattern is symmetrical and only the half above the nozzle axis need be considered. The wave pattern at any point in the kernel is not influenced by the wave pattern downstream of that point. Consequently, the kernel for any corner less than the maximum of 51.16° can be obtained from the kernel for 51.16° by neglecting the characteristics of value greater than the desired corner angle. This principle is illustrated in figure 2(c).

The bounding characteristic separating zone I from zone II (fig. 2(b)) is designated as  $\psi_f^+$ . The points of intersection of the  $\psi^-$  characteristics with the  $\psi_f^+$  characteristic, and the slopes of the  $\psi^-$  characteristics at these points, are all that is required to determine the nozzle contour.

The constructed kernel for M = 10.00 provided data for the design of nozzles for final Mach numbers  $M_f$  from 1.20 to 2.00 in increments of 0.20 and from 2.00 to 10.00 in increments of 1.00. The coordinates  $\left(\frac{x}{A_t/2}, \frac{y}{A_t/2}\right)$  of the points of intersection of the bounding characteristic  $\psi_f^+$  with the  $\psi^-$  characteristics are tabulated with other pertinent data in table I.

For Mach numbers up to 4.00, a kernel of 12-inch half throat height  $A_{\rm t}/2$  was graphically developed and for Mach numbers from 5.00 to 10.00 a half throat height of 6 inches was used. For the

6-inch kernel, however, the scale was reduced at intervals as the height of the kernel increased in order to maintain the construction within the physical limit of the drawing board. This scale reduction accounts for the decreasing number of decimal places for the coordinates in table I in the high Mach number range. Turningangle increments in  $\psi^+$  and  $\psi^-$ , as given in the preceding table, were used for both kernels. Construction was performed with a drafting machine capable of setting to  $\pm 2.5$  minutes.

#### Wall Contour

An expansion wave incident on a channel wall will, in general, require that a secondary wave be emitted at the point of incidence in order to keep the flow against the wall. If the wall is curved in the way a streamline would be turned under the influence of the incident wave, however, no secondary wave arises to keep the flow along the wall. This method of suppression of secondary waves is the principle used to obtain uniform wave-free flow in the test section. The graphical construction is required to locate the point of incidence of the waves on the nozzle wall. The difference in value of the characteristics bounding the incident wave gives the change of wall inclination required to suppress secondary waves (fig. 2(d)); that is, for the upper wall,

$$\Delta \alpha = \Delta \psi^{-} \tag{3a}$$

or for the lower wall,

$$\Delta \alpha = \Delta \psi^{+} \tag{3b}$$

where  $\Delta\alpha$  is the required change of wall inclination. The accuracy of the wall contour obtained improves as the number of characteristics drawn to represent the incident expansion wave is increased. Only the upper nozzle wall need be developed if the nozzle is symmetrical about the center line.

Symmetrical two-dimensional sharp-edge-throat nozzles are produced by making the angle of the turn at both walls at the throat equal in magnitude. If  $\phi$  represents the angle of turn for the upper and lower walls, the downstream characteristics  $\psi_{\mathbf{f}}^{+}$  and  $\psi_{\mathbf{f}}^{-}$  that bound the respective expansion waves will have this value. Because of the symmetry of the wave pattern about the nozzle axis, a  $\psi^{+}$  characteristic will intersect a  $\psi^{-}$  characteristic of the same magnitude at the nozzle axis. In particular, the  $\psi_{\mathbf{p}}^{+}$  and  $\psi_{\mathbf{p}}^{-}$ 

characteristics intersect on the nozzle axis (fig. 2(c)). The flow along the streamline on the nozzle axis will have the final Mach number  $M_f$  at the intersection of these bounding characteristics. From equation (1),

$$\Psi_{f}^{+} + \Psi_{f}^{-} = \Psi_{f} = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \frac{\sqrt{M_{f}^{2}-1}}{\sqrt{\frac{\gamma+1}{\gamma-1}}} - \tan^{-1} \sqrt{M_{f}^{2}-1}$$

Because  $\psi_f^+$  and  $\psi_f^-$  are equal in magnitude and represent the angle through which each wall is turned at the throat,

$$\varphi = \psi_{f}^{+} = \psi_{f}^{-} = \frac{\psi_{f}}{2} = \frac{1}{2} \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \frac{\sqrt{M_{f}^{2-1}}}{\sqrt{\frac{\gamma+1}{\gamma-1}}} - \tan^{-1} \sqrt{M_{f}^{2-1}}$$
(4)

Equation (4) gives the value of the wall angle at the throat that corresponds to the desired test-section Mach number  $M_{f}$ . These values are presented in table II, columns 1 and 2.

The method of using the kernel that is schematically shown in figure 3(a) to obtain the nozzle-wall contour of two-dimensional sharp-throat nozzles of minimum length (fig. 3(b)) is illustrated by application to a specific problem. Assume that it is desired to design a nozzle of this class with a test-section Mach number of 4.00 and a throat height of 6 inches.

The throat-corner angle  $\phi$  and the value of the downstream bounding characteristics  $\psi_1^+$  are obtained from equation (4) or table II, column 2, with  $M_P$  equal to 4.00, column 1:

$$\varphi = \psi_f^+ = \psi_f^- = 32.89^\circ$$

The wall contour is obtained by plotting the zone II characteristics of the  $\psi^-$  set (fig. 3(b)), which are straight lines that make the angle  $\lambda^-$  with the nozzle axis at the intersection of the  $\psi^-$  characteristics and the bounding characteristic  $\psi_f^+$ . All that is required to obtain the zone II plot are the coordinates of the points

of intersection of the  $\psi^-$  set of characteristics with the bounding characteristic  $\psi_f^+$  and the local slopes  $\lambda^-$  of the  $\psi^-$  characteristics. Columns 4 and 5 of table I give the coordinates of intersection in terms of the half throat height  $A_\pm/2$  and column 6 gives the angle of inclination  $\lambda^-$  of the  $\psi^-$  characteristic at the intersection. For example, the  $\psi^-$  = 12.00° characteristic intersects the  $\psi_f^+$  characteristic at  $\frac{x}{A_\pm/2} = 3.453$  and

 $\frac{y}{A_t/2}$  = 1.256, which gives x = 10.359 and y = 3.768 for a nozzle  $\frac{x}{A_t/2}$ 

of 6-inch throat. The inclination of the  $\psi^-$  characteristic in zone II is  $\lambda^- = 42.14^\circ$ . The complete plot of the zone II characteristics has the form schematically illustrated in figure 4(a).

Construction of the nozzle wall starts at the nozzle throat with a straight-line segment ab (fig. 4(b)) that makes the angle with the nozzle axis  $\varphi = 32.89^{\circ}$ , which was previously computed for  $M_{C} = 4.00$ . At the intersection of the nozzle wall with the first  $\psi^-$  characteristic ( $\psi_1^- = 0.01^\circ$ ), the inclination of the wall is reduced according to equation (3a) by an amount  $(\psi_1^- - \psi_0^-)$  corresponding to the angle through which the flow is turned clockwise by the expansion wave between  $\psi_0$  and  $\psi_1$ . As previously discussed, no wave emission occurs at the wall turned in this way. At every intersection of the wall with a characteristic, the wall inclination to the nozzle axis is reduced by the angle of turning produced by the wave between  $\psi_n$  and  $\psi_{n-1}$ . The angle of the wall  $\alpha$  at each characteristic is given in table I, column 7. For example, at point b,  $\psi^- = 0.01^{\circ}$  and  $\alpha = 32.88^{\circ}$ ; similarly at point c,  $\psi^- = 0.04^{\circ}$  and  $\alpha = 32.85^{\circ}$ . When the sequence of straight-line segments representing the nozzle wall is completed, a smooth curve approximating the shape of the sequence of straight lines is taken as the effective nozzle-wall contour. The accuracy of the final wall contour increases with the number of characteristics used to represent the expansion waves from the wall corners at the nozzle throat.

An averaging method for attaining a contour that is closer to the true contour will be described for a nozzle with a test-section Mach number  $M_{\Gamma}$  of 4.00 as an example, as shown in figure 4(b). As before, construction starts at the nezzle throat with a straight line ab making the corner angle with the nozzle axis  $(\varphi = 32.89^{\circ})$ . Line ab is then bisected by point c, and line cd is drawn at the wall angle  $\alpha = 32.88^{\circ}$ , corresponding to  $\psi^{-} = 0.01^{\circ}$ , (table I, column 7) until it intersects the  $\psi^{-} = 0.04^{\circ}$  characteristic.

Point B, the wall coordinate point lying along  $\psi^-=0.01^\circ$ , is located by the intersection of line cd and  $\psi^-=0.01^\circ$ . Line Bd is then bisected by point e, and line eg is drawn at the wall angle  $\alpha=32.85^\circ$  corresponding to  $\psi^-=0.04^\circ$ . Point D is located by the intersection of line eg and  $\psi^-=0.04^\circ$ . The preceding process is continued until the design is complete. The nozzle contour is taken as the smooth curve through points a,B,D, . . ., tangent to construction lines ab, cd, eg, . . .

The test-section height of the nozzle (numerically equal to  $A_f$ ), which is obtained by either of the graphical processes described, should be related to the throat height by the expression

$$\frac{A_{f}}{A_{t}} = \frac{1}{M_{f}} \left( \frac{1 + \frac{\gamma - 1}{2} M_{f}^{2}}{\frac{\gamma + 1}{2}} \right)^{\frac{\gamma + 1}{2(\gamma - 1)}}$$
(5)

These area-ratio values are presented in table II, column 4. For example, for  $M_{f} = 4.00$ ,  $\frac{A_{f}}{A_{t}} = 10.719$ .

The design of a nozzle that has a Mach number intermediate between values given in table I requires the determination of the shape of the  $\psi_f^+$  characteristic of the kernel corresponding to the desired Mach number. This design is accomplished by using the coordinates of the  $\psi_f^+$  characteristic given in table I that are closest to the desired Mach number and then establishing by construction the points of intersection of the  $\psi_f^+$  characteristics that correspond to the desired Mach number with the  $\psi^-$  characteristics, as shown in figure 5. For example, the kernel for M = 4.30 is established with the kernel for M = 4.00 as a base.

The bounding characteristic  $\psi_f^+$  and the zone II plot of  $\psi_f^-$  characteristics for  $M_p=4.00$  are established as previously described. These characteristics are dashed in figure 5. The bounding characteristic and the zone II plot of  $\psi_f^-$  characteristics for  $M_p=4.30$  are established according to the following procedure:

The value of the bounding characteristic  $\psi_f^+$  for  $M_f = 4.30$  is obtained from equation (4) or table II, column 2,  $(\psi_f^+ = 34.77^\circ)$ .

The angle that the  $\psi_1^+$  characteristic makes with the nozzle axis at any point is designated  $\psi^+$  (fig. 5) and is determined by the relation

$$\lambda^+ = \beta - \theta \tag{6}$$

where  $\beta$  is the Mach angle determined by the local Mach number corresponding to the local equivalent Prandtl-Meyer turning angle  $\psi$ , given by equation (1), and  $\theta$  is the angle of inclination of the flow to the nozzle axis, given by equation (2). (Note that positive values of  $\lambda^+$  are drawn with a negative slope.) Thus at point A at the throat (fig. 5):

from equation (1),

$$\Psi = \Psi^+ + \Psi^- = 34.77 + 0 = 34.77^{\circ}$$

from table II, columns 3 and 5, for  $\psi = 34.77$ ,

$$\beta = 25.53^{\circ}$$

from equation (2),

$$\theta = \psi^+ - \psi^- = 34.77 - 0 = 34.77^\circ$$

Consequently,

$$\lambda^{+} = \beta - \theta = 25.53 - 34.77 = -9.24^{\circ}$$

The negative sign indicates that  $\lambda^+$  is drawn with positive slope, as shown at point A of figure 5. The bounding  $\psi_f^+$  characteristic is drawn at the angle  $\lambda^+ = -9.24^\circ$ , until it intersects the first  $\psi^-$  characteristic  $\psi^- = 0.01^\circ$  at point B. At point B the new  $\lambda^+$  value for  $\psi_f^+$  is determined by repeating the aforementioned procedure using  $\psi^+ = 34.77^\circ$  and  $\psi^- = 0.01^\circ$ . The  $\psi_f^+$  characteristic is drawn at this new  $\lambda^+$  value until it intersects the next  $\psi^-$  characteristic  $\psi^- = 0.04^\circ$ . The slope of the  $\psi^-$  characteristic at point B,  $\lambda^-$ , is determined by the relation (fig. 5)

$$\lambda^- = \beta + \theta \tag{7}$$

with the same values for  $\beta$  and  $\theta$  as were used to determine  $\lambda^+$ . In this manner the entire zone II plot of  $\psi^-$  characteristics is obtained for  $M_P = 4.30$ .

The wall contour is then developed by the method previously described for  $M_{\Gamma}=4.00$ . The entire procedure is expedited if columns 1 to 3, 6, and 7,  $\phi$  and  $\psi_{\Gamma}^{+}$ , of table I, and  $\lambda^{+}$  are determined for the  $\psi_{\Gamma}^{+}$  characteristic for M=4.30 before the drawing is initiated.

# Nozzle Length

The nozzle length from the throat to the test section may be calculated from the length of the kernel and the projection of the last characteristic on the nozzle axis, as shown in figure 3(b). The projection may be determined from the final Mach angle and the final area ratio. The expression for the ratio of the nozzle length to the nozzle test-section height

$$\frac{L}{A_{f}} = \left(\frac{l_{k}}{A_{t}} + \frac{A_{f}}{2A_{t} \tan \beta}\right) \frac{A_{t}}{A_{f}}$$

is plotted in figure 6 for Mach numbers up to 10.

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#### REFERENCES

- 1. Pinkel, I. Irving: Equations for the Design of Two-Dimensional Supersonic Nozzles. NACA RM No. E8B02, 1948.
- 2. Puckett, A. E.: Supersonic Nozzle Design. Jour. Appl. Mech., vol. 13, no. 4, Dec. 1946, pp. A265-A270.
- Shapiro, Ascher H., and Edelman, Gilbert M.: Method of Characteristics for Two-Dimensional Supersonic Flow Graphical and Numerical Procedures. Jour. Appl. Mech., vol. 14, no. 2, June 1947, pp. Al54-Al62.
- 4. Taylor, G. I., and Maccoll, J. W.: The Two-Dimensional Flow Around a Corner; Two-Dimensional Flow Past a Curved Surface.

  Vol. III of Aerodynamic Theory, div. H, ch. IV, secs. 5-6,
  W. F. Durand, ed., Julius Springer (Berlin), 1935, pp. 243-249.

1	2	3	4	5	6	7	1	2	3	4	5	6	7
Ţ-	Ŧ	В	x	У	λ-	a	¥-		В		7	1-	α
(deg)	(deg)	(deg)	At/2	A <sub>t</sub> /2	(deg)	(deg)	(deg)	(deg)	(deg)	At/2	A <sub>t</sub> /2	(deg)	(deg)
1	¥ŗ,	1.20;	• and	Yr+,	1.80°		0.31 .37	7.81 7.87	47.41 47.32	0.557	0.525	54.60 54.45	7.19 7.15
0	1.80	62.91	0	1.000		1.80	11 .46	7.96	47.17	.600	489	54.21	7.04
.01	1.81	62.87 62.72	.213 .277	.614		1.79	.55	8.05	47.05	.622	.471	53.98	6.95
.07	1.87	62.59	305	.448		1.73	.64 .73	8.14 8.25	46.89	643	.454 .439	53.75 53.52	6.86
.10	11.90	62.45	.328	.408		1.70	.82	8.32	46.61	679	425	53.29	6.68
.13	1.95 1.96	62.32 62.18	.343 .356	.380 .357	63.99 63.82	1.67	1.00	8.41 8.50	46.47	.694 .708	.412 .399	53.06 52.83	6.59
.19	1.99	62,05	.368	.336	63.66	1.61	[  1.20	8.70	46.03	738	375	52.55	6.50
		61.78	.386	.305	63.33	1.55	1.40	8.90	45.73	.764	.353	51.85	6.10
.31 .37	2.11	61.52 61.28	.404	.274 .253	63.01 62.71	1.49	1.60	9.10 9.30	45.43	.787 .810	.333	51.33 50.84	5.90
		60.89	433	223	62.23	1.43	2.00		45.14 44.85	828	300	50.35	5.70 5.50
.55	2.35	60.53	.448	.198	61.78	1.25	2.40	9.90	44.32	.867	.268	49.42	5.10
	2.44	60.18	.463	.176	61.34	1.16	2.80	10.50	43.78	.903	.239	48.48	4.70
		59.84 59.50	.475 .487	.155 .136	60.91 60.48	1.07	5.20   5.60	10.70	43.28 42.80	.934 .963	.213	47.58	4.50 5.90
91	2.71	59.18	498	118	60.07	.89	114.00	11.50	42.30	992	.167	45.80	5.50
1.00	2.80	58,86	.507	.103	59.66	•80	4.50	12.00	41,72	1.022	142	44.72	5.00
		58.18 57.57	.527 .543	.072 .046	58.78 57.97	.60	5.00	12.50	41.15	1.054	117	43.65	2.50
		56.98	.559	.022	57.18	.20	6.00			1.115	093	42.60 41.55	2.00 1.50
1.80		56,40	.574	0	56,40	0	6.50	14.00	39.53	1.145	.045	40.53	1.00
	Me,	1.40;	<pre>and</pre>	¥.+,	4.500		7.00		39.04	1.172	0.022	39.54	•50
0		53.99	0	1.000		4.50	7.50			1.202 • and		38.54	0
		53.97	.257	.698		4.49	<u> </u>					0.00-	10.55
.04	4.57	53.89 53.82	.335 .369	.607	58.35 58.25	4.46	0.01	10.36	43.70	320	1.000	54.04	10.35
.10	4.60	55,75	.397	.535	58.15	4.40	.04	10.40	43.65	1 .417	.725	53.97	10.52
.13	4.65	53.68	.417	.513	58.05	4.37	.07	10.45	43.61	.460	.697	53.90	10.29
.16	4.69	53.60 53.55	.432 .447	.495 .477	57.94 57.84	4.34	.10	10.46	43.53	.494 .517	675 659	53.83 53.76	10.25
•25	4.70	55.58	.469	.452	57.65	4.25	.13 .16	10.52	43.50	.537	.646	53.70	10.20
.31	4.81	55.23	.491	427	57.42	4.19	.19	10.55	45.46	-557	.633	53.63	10.17
•37 •46	4.87 4.96	53.08 52.87	.507 .527	407 384	57.21 56.91	4.15	.25 .31 .37	10.61	45.39 45.32	583 612	.615 .597	53.50 53.37	10.11
.55	5.05	52.66	.547	.363	56.61	3.95	.37	10.67	43.24	632	.583	53.23	9.99
		52.46 52.26	.564 .580	.342	56.52 56.03	3.85	.46 .55	10.82	43,14 43,03	.658 .682	.567 .550	53.04 52.84	9.90
		52.06	.594	.308	55.74	3.77 3.69	.64		42.92	704	535	52.64	9.81 9.72
.91	5.41	51.86	.608	.293	55.45	3.59	.73	11.09	42.81	.725	.522	52.44	9.63
1.00		51.66	.620	.279	55.16	3.50	82	11.18		.743	.510	52,24	9.54
		51.25 50.83	.645 .667	.252 .227	54.55 53.93	5.30 5.10	1.00	11.27 11.36	42,59 42,47	.762 .777	.498 .488	52.04 51.83	9.45
1.60	6.10	50.43	. 687	205	53.33	2.90	1.20	11.56	42.23	.811	.467	51.39	9.16
		50.05	.706	.185	52.75	2.70	1.40	11.76 11.96	42.00	.841	.447	50.96	8.96 8.76
2.00	6.90	49.66 48.93	.722 .752	.167 .135	52.16 51.03	2.50 2.10	1.60 1.80	12.16	41.77	.867 .892	.430 .413	50.53 50.10	8.56
2.80	7.30	48.24	783	.101	49.94	1.70	2.00	12.36	41.31	.912	.400	49.67	8.36
3.20	7.70	47.58	.808	.075	48.88	1.30	2,40	12.76	40.86	957	.371	48.82	7.96
		46.95 46.33	.832 .856	.050 .025	47.85 46.83	.90 .50	2.80	13.16 13.56	40.42 50.00	.997 1.034	.345 .321	47.98 47.15	7.56 7.16
4.50	9.00	45.57		0	45.57	0.00	3.60	13.96	39.57	1.067	.300	46.33	6.76
			• and	¥.+,	7.500		14.00	14.36	39,18	1.102	.277	45.54	6.36
0 1	7.50	47.90	0	1.000		7.50	4.50 5.00	14.86 15.36	38.21	1.137	.254 .231	44.54 43.57	5.86 5.36
.01	7.51	47.88	_292	.751	55.37	7.49	5.50	15.86 16.36	37.76	1.208	208	42.62	4.86
.04	7.54	47.88 47.84 47.79 47.74	.381	.676	55.37 55.30	7.46	6.00	16.36	37.30	1.247	183	41.66 40.72	4.36
-10	7.57 7.60	47.74	.421 .451	.642 .616	55.22 55.14	7.43 7.40	6.50 7.00	16.86 17.36	36.42	1.282	.162	39.78	3.86 3.36
13	7.00	47,09	.472	.597	55.06	7.37	7.50	17.86	36.00	1.350	.117	38.86	2.86
- 36	7.66	47.64	.492 500	.582	54.98	7.34	8.00	18.36	35.59	1.379	.097 .055	37.95 36.16	2.36
*18	7.69 7.75	47.60 47.50	508 533	.567 .546	54.91 54.75	7.81 7.25	9.00	19.36 20.36	34.80 34.04	1.443	012	34, 40	1.36
							10,36	20.72	33.76	1.528	0	33.76	0.00

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T 1	2	3	4	5	6	7	1	2	3	4	5	6	7
¥-	¥	В	×	J	λ-	4	¥-	¥	β	x	7	λ-	g
(deg)	(deg)	(deg)	At/2	A <sub>t</sub> /2	(deg)	(deg)	(deg)	(deg)	(deg)	A <sub>t</sub> /2	A <sub>t</sub> /2	(deg)	(deg)
	M <sub>e</sub> , 2	.00;	and i	r*, 18	5.19°			26.88 27.28	29.48		.850	51.96	
0	25 70	40.38	0	1 000		13.19		27.68 28.08				51.32 50. <b>69</b>	
	13.20		.345		53,56	13.18		28.48		1.555	826	50.07	21.28
	13.23		449		53.50	13.15		28.88		1.616		49.44	
10	13.26	40.27	.496 .533		53.43	13.12 13.09		29.38 29.88		1.678 1.745	800	48.67 47.91	19.88
.13	13.32	40.25	<b>.</b> 557	.715	53.31	13.06	5.50	30.38	27.75	1.812	.792	47.13	19.38
-16	13.35 13.38	40.22	.580 .501		53.25 53.18	13.03 13.00		30.88 31.38		1.883		46.35	
-25	13.44	40.12	.630	.678	53.06	12.94	7.00	31.38	26.96	2.015	.761	44.84	17.88
.31	13.50	40.05	.660		52.93	12.88		32.38			.750	44.09	17.38
	13.56		.683		52.81 52.62	12.73		32.88 33.88				41.85	
.55	13.74	39.90	.737	.622	52.44	12.64	10.00	34.88	25.49	2.417	.693	40.37	14,88
-64	13.83	39.71	.762 .784		52.26 52.07	12.55 12.46	12.00	35.88 36.88	25.04	2.554 2.695	667	38.92 37.47	13.88
	14.01		804		51.89	12.57	13.00	37.88	24.12		.611	36.00	11.88
	14.10		.824		51.71	12.28		38.88		2.982		34.57	10.88
	14.19		.842 .878		51.53 51.14	12.19 11.99		39.88 40.88		3.138 3.289		33.15 31.73	9.88
	14.59		.911	.533	50.74	11.79		41.88				30.31	7.88
	14.79		.939	.519	50.34	11.59		42.88				28.91	6.88
2.00	14.99	38.37	.938		49.94 49.56	11.39 11.19	27.00	45.88 45.88	50.88	4-167		27.52 24.76	5.88 5.88
2.40	15.59	38.00	1.041	.467	48.79	10.79	23.00	47.88	20.14	4.584	.144	22.02	1.88
2.80	15.99 16.39	37.64	1.088		48.03	10.39	24.88	49.76				19.47	10 [
3.80	16.79	36.92	1.128		47.26 46.51	9.99 9.59	1	M <sub>r</sub> , 4	ŀ.00; ¢	and T	r <sup>+</sup> , 32	.89°	i
4.00	17.19	36.65	1.206	.382	45.84	9.19	0	32.89	26.46	0			32.89
	17.69				44.83 43.92	8.69 8.19	-01	32.90 32.93	26.45	.507 .661	1.056	59.33 59.28	32.88
	18.69				43.02	7.69		32.96		.731	1.080	59.24	
6.00	19.19	34.93	1.373	.295	42.12	7.19	.10	32.99	26.40	•785	1.086	59.19	
7.00	19.69	34.55	1.413		41.24	6.69 6.19		33.02 33.05		.822 .857		59.15 59.10	32.73
	20.69				39.55	5.69	.19	33.08	26.36	887	1.097		
	21.19				38.60	5.69 5.19 4.19 5.19 2.19	.25	33.14		.931			32.64
	22.19				36.89 35.20	5.19	-37	33.20 33.26	26.30	977		58.88 58.79	
	24.19			.092	33.55	2.19	.46	33.35	26.22	1.053	1.116	58.65	32.43
	25.19				31.93	1.19	.55				1.120		32.34
	26.19		1.918		30.31	0.19		35.53			1.125		32.25 32.16
							.82	33.71	26.04	1.204	1.132	58.11	52.07
<u> </u>			and Y	1.000		24.88	.91		26.01				31.98
0.01	24.88 24.89	30.92	.439	953	55.79	24.87	1.20	34.09	25.86	1.326	1.138		51.69
•04	24.92	30.90	.573	.939	55.74	24.84	1.40	34.29	25.76	1.380	1.150	57.25	31.49
-07	24.95 24.98	30.87	.633		55.69 55.65	24.81 24.78		34.49 34.69					31.29
	25.01				55.60	24.75	2.00	34.89	25.48	1.522	1.164	56.37	30.89
-16	25.04	30.82	.742	.921	55.54	24.72	2.40	35.29	25.30	1.608	1.172	55.79	30.49
.19	25.07 25.13	30.81	.768 .805		55.50 55.40	24.69 24.65	1 3.20	36.09	24.95	1.772	11.187	154.64	129.69 I
.31	25.19	30.73	.845	.910	55.30	24.57	3.60	36.49	24.77	1.849	1.192	54.06	[29.29]
.37	25.25	30.69	.875	.906	55.20	24.51	1 4.00	36.89	124.58	11.927	11.199	153.47	128.291
	25.34 25.43				55.06 54.91	24.42 24.33	5.00	37.89	24.11	2.099	1.205 1.212	52.00	27.89
.64	25.52	30.53	.980	.895	54.77	24.24	1 5.50	38.39	23.90	2.187	1.217	151.29	27.39
.73	25.61	30.47	1.010		54.62	24.15	6.00	38.89	23.68	2.285	1.223	50.57	26.89
91	25.70 25.79	30.36	1.065		54.48	24.06 23.97	7,00	39.89	23.26	2.467	1.233	49.15	25.89
1.00	25.88	30.30	1.088	.883	54.18	25.88	7.50	40.39	23.05	2.563	1.237	48.44	25.39
1.20	26.08	30.18	1.138		53.86	25.68	8.00	40.89	22.84	2.650	1.241	47.73	24.89
1.60	26.28 26.48	29.94	1.222		53.54 53.22	23.48 23.28	10.00	42.89	22.02	3.039	1.247	44.91	22.89
	26.68				52.96	23.08	11.00	43.89	21.63	3.242	1.255	43.52	21.89



TABLE 1. - DETAILED MOZZLE DESIGN PARAMETERS - Continued  $\begin{bmatrix} \gamma = 1.400 \end{bmatrix}$ 

1	2	3	4	5	6	7	1	2	3	4	5	6	7
¥-	Ţ	β	x	7	λ_		<u>v</u> -	Ţ				λ-	
(deg)		(deg)	At/2	At/2	(deg)	(deg)	(deg)		β (deg)	At/2	$\frac{\mathbf{J}}{\mathbf{A_t/2}}$	(deg)	(deg)
	¥ .	4.00:	φ and ¥	+. 32	.890		25.00		15.13		2.083	28.59	15.46
	_f,		7	f, on			27.00		14.57	12.877			9.46
12.00	44.89	21.25	3.453	1.256	42.14	20.89	31.00			14.827	1.733		7.46
15.00	45.89	20.87	3.669		40.76	19.89	33.00	71.46	12.93	17.187	1.460	18.39	5.46
14.00				1.252		18.89	35.00	73.46	12.41	19.720 23.027	1.100	15.87 13.36	3.46 1.46
15.00				1.235		17.89 16.89	37.00			25.867		11.54	0
17.00				1.221	35.31	15.89	122720			o and Y			
18.00			4.932	1.204	33.98	14.89	<del> </del>						40 40
19.00	51.89	18.73		1.183		13.89 11.89	0.01	42.49	22.20		1.217	64.66	42.48
23.00				1.050		9.89	04				1.283		
25.00	57.89	16.79	7.445	.939	24.68	7.89	.07	42.55	22.17	.860	1.313	64.58	
27.00					22.08	5.89		42.58			1.337	64.54	
29.00			9.518 10.770		19.48	3.89 1.89	.13			1.010	1.353	64.45	42.35 42.32
			12.179		14.48	0	.19				1.380	64.41	42.29
			e and T				.25	42.73	22.09	1.100	1.400	64.32	42.23
							.31	42.79		1.147		64.23	
		23.88			62.32	38.46	.37	42.85 42.94			1.433	64.15	42.11 42.02
	38.47 38.50		.560 .727	1.143		38.45 38.42		43.03		1.247	1.470		41.93
	38.53	23.85	800			58.39		43.12		1.347		63.77	41.84
	38.56		.863	1.220	62.20	38.56	.73	43.21	21.90	1.387	1.503	63.65	41.75
.13	38.59			1.233		38.33		43.30		1.427		63.53	41.86
	38.62 38.65		.940 .973	1.240		38.30 38.27		43.39 43.48		1.467	1.530	63.40 63.28	41.48
	38.71		1.027	1.263		38.21		43.68		1.580	1.570		41.28
.31	38.77	23.74	1.070	1.273	61.89	38.15	1.40	43.88	21.64	1.647	1.593	62.72	41.08
	38.83				61.80	38.09		44.08		1.703	1.613	62.44	
	38.92 39.01		1.160	1.293	61.67 61.55	38.00 37.91		44.28 44.48		1.770	1.637	62.16 61.88	40.68
	39.10					37.82		44.88		1.937		61.34	40.08
.73	39.19	23.56	1.290	1.327	61.29	37.73	2.80	45.28	21.11	2.047	1.750		39.68
	39.28		1.330	1.337		37.64		45.68		2.153	1.767	60.23	39.28
	39.37 39.46		1.363	1.347	61.03	37.55 37.46		46.48		2.253 2.363	1.800	59.68 59.14	38.88 38.48
	39.66			1.370		37.26		46.98		2.477		58.45	37.98
	39.86			1.387		37.06		47.48		2.600	1.910		37.48
	40.06			1.397		36.86		47.98		2.727	1.947		
	40.26		1.640	1.413		36.66		48.48		2.867 2.997	2.027	56.40	36.48
	40.46		1.690	1.423		36.46 36.06		48.98		3.127		55.05	35.98 35.48
	41.26		1.887	1.470		35.66		49.98		3.273		54.37	34.98
3.20	41.66	22.53	1.980	1.490	57.79	35.26	8.00	50.48	19.22	3.403	2.140	53.70	34.48
	42.06		2.070	1.510		34.86		51.48		3.690	2.217		33.48
	42.46 42.96		2.167 2.267	1.533	56.67 55.96	34.46 33.96		52.48 53.48		3.977 4.293	2.290		32.48 31.48
	43.46		2.373	1.577	55.27	33.46		54.48		4.620	2.440		
5.50	43.96	21.61	2.480	1.597	54.57	32.96	[13.00]	55.48	17.56	4.983	2.517	47.04	29.48
	44.46		2.607		53.67	32.46		56.48		5.357	2.597		28.48
	45.46	21.23	2.710 2.827	1.643	53.19 52.50	31.96 31.46		57.48 58.48		5.770 6.187	2.673	44.41	27.48 26.48
7.50	45.96	20.85	2.945	1.687	51.81	30.96	17.00	59.48	16.32	6.667	2.830		
		20.67	3.057	1.707	51.13	30.46	18.00	60.48	16.02	7.150	2.903	40.50	
	47.46		3.303	1.750	49.76	29.46	19.00	61.48	15.72	7.683	2.977	39.20	23.48
10.00	40.46	19.57	3.813	1.790	47.03	28.46 27.46	23.00	65.49	14.57	8.897 10.337	3.267	34.05	19.48
12.00	50.46	19.22	4.083	1.867		26.46	125.001	67.48	14.01	12.000	3.377	31.49	17.48
13.00	51.46	18.89	4.377	1.903	44.35	25.46	27.00	69.48	13.47	14.010 16.400	3.480	28.95	15.48
14.00	52.46	18.54	4.680	1.937	43.00	24.46	29.00	71.48	12.93	16.400	3.533	26.41	13.48
15.00				1.967		23.46	31.00	75.48	12.41	19.200 22.667	3.527	23.89	11.48
16.00			5.715	1.997	39.02	22.46 21.46	35,00	77.40	11.40	26.547	3.223	21.08	7.48
18.00	56.46	17.25	6.090	2.047	37.71	20.46	37.00	79.48	10.90	31.61	2.80	16.38	5.48
19.00	57.46	16.93	6.500	2.067	36.39	19.46	39.00	81.48	10.41	26.547 31.61 37.93	2.13	13.89	3.48
21.00				2.097		17.46	141.00	83.48	9.95	46.03	1.05	11.43	1.48
23.00	61.46	16.72	8.497	2.103	31.18	15.46	42.48	84.96	9.59	52.55	0	9.59	0



TABLE I. - DETAILED NOZZLE DESIGN PARAMETERS - Continued  $\gamma = 1.400$ 

1	2	5	4.	5	6	7	T 1	2	3	4	5	6	7
¥-	¥	β	x	y	λ-	-	¥-	¥	β	x	7	λ-	<u>.</u>
(deg)	(deg)	(deg)	1-12	At/2	(deg)	(deg)	1	(deg)		At/2	$\frac{1}{4t/2}$	(deg)	(deg)
	ĸ <sub>r</sub> ,	7.00;	e and Y	<sup>+</sup> , 45.	49°					φ and Y			
0	45,49	21.02	0	1.000		45.49	0	47.82	20.17 20.16	0 -653	1.000	67.96	47.81 47.80
.01	45.50	21.02	.627	1.280	86.50	45.48	-04	47.85	20.15	.853	1.443	67.92	47.77
	45.53 45.56			1.367					20.14		1.487		47.74
	45.59			1.440					20.13	1.063	1.527		47.71 47.68
.13	45.62	20.97	1.020	1.460	66.33	45.36	.16	47.97	20.10	1.107	1.573	67.75	47.65
.16	45.65 45.68	20.96		1.480			.19	48.00	20.09	1.147	1.593	67.71 67.63	47.62 47.56
25	45.74	20.93	1.157	1.523	66.17	45.30 45.24	.31	48.12	20.04	1.260	1.653	67.54	47.50
.31	45.74 45.80	20.91	1.210	1.543	66.09	45.24 45.18 45.12 45.03 44.94 44.85 44.76 44.67 44.58	.57	48.18	20.04 20.03	1.313	1.680	67.47 67.34	47.44
.37	45.86 45.95	20.89		1.567		45.12	-46	48.27	19.90	1.367	1.710	67.34	47.35
-46 -55	46.04	20.81	1.367	1.590	65.75	44.94	.64	48.36	19.95	1.480	1.767	67.22 67.10	47.26 47.17
.64	46,13	20.78	1.420	1.637	65.63	44.85	.73	48.54	19.89	1.527	1.790	67.10 66.97	47.08
	46.22		1.463	1.657	66.51	44.76	.82	48.63	19.88	1 7 572	ידינים ורו	IEE OEI	46.99
82	46.31 46.40	20.72	1.850	1.677	65.39 65.27	44.67	31	48.72	19.80	1.617	1.867	66.73	46.90 46.81
1.00	46.49	20.65	1.587	1,713	65.14	44.49	1.20	49.01	19.83 19.80 19.73 19.66	1.743	1.903	66.73 66.61 66.34 66.07	46.61
	46.69		1.670	1.750	64.86	44.29	1.40	49.21	19.66	1.820	1.940	66.07	46.41
	46.89 47.09		1.743	1.780	64.59	44.09	T* OO	49.3T	TA.0A	T-001	2.013	65.80	46.21
1.80	47.29	20.35	1.880	1.840	64.04	43.89 43.69	2.00	49.81	19.52 19.45	2.027	2.043	65.26	46.01 45.81
2.00	47.49	20.28	1.940	1.867	63.77	43.49	2.40	50.21	19.31	2.157	2.110	64.72	45.41
	47.89			1.920			2.80	50.61	19.18		2.173		45.01
	48.29 48.69			1.970 2.017			3.80	51.41	19.06	2.533	2.230	63.12	44.61 44.21
3.60	49.09	19.70	2.410	2.067	61.59	41.89	4.00	51.81	18.77	2.663	2.353	62.58	43.81
4.00	49.49	19.56 19.38 19.21 19.06	2.533	2.117	61.05	41.49	4.50	52.31	18.60	0.00#	2.417	61.91	43.31
4.50	49.99	19.38	2.663	2.167	50.37	40.99 40.49	5.00	52.81	18.44	2.953	2.487 2.553	61.25	42.81 42.31
5.50	50.99	19.06	2.940	2.277		39.99	6.00	53.81	18.44 18.28 18.11 17.94	3.283	2.653	59.92	41.81
6.00	27.48	18.84	3.100	2.337	58.36	39.49	6.50	54.31	17.94	3.447	2.707	59.25	41.31
6.50	51.99	18.70	3.247	2.390	57.69	38.99	7.00	54.81	17.78	3.613	2.777		40.81
7.50	52.49 52.99	18.38	3.555	2.445 2.503	56.37	38.49 37.99			17.62 17.46		2.903	57.93	40.31 39.81
8.00	53.49	18.21	3.710	2.557	55.70	37.49	9.00	56.81	17.14	4.300	3.060	55.95	38.81
	54.49		4.023	2.667	54.36	36.49	10.00	57.81	16.82	4.673	5.207	54.63	37.81
10.00	55.49	17.55	4.707	2.773	53.04	35.49 34.49	12.00	58.81	16.52	5.527	3.360	53.33	36.81 35.81
12.00	57.49	16.92	5.113	3.007	50.41	33.49	13.00	60.81	16.52 16.22 15.92 15.62	6.003	3.360 3.520 3.683	50.73	34.81
13.00	58.49	16.61	5.537	3.007 5.130	49.10	32.49	14.00	61.81	15.62	6.517	3.857	49.40	33.81
14.00	59.49	16.31	5.983	3.253 3.383	47.80	31.49	I TO " OO	02.01	10.02	7.000	4.037	48.13	32.81 31.81
16.00	61.49	18.71	6.980	3.510	45.20	30.49 29.49	17.00	64.81	15.04 14.76	8.327	4.410		30.81
17.90	62.49	15.41	7.553	3.643	43.90	28.49	18.00	65.81	14.48	9.023	4.603	44.29	29.81
18.00			8.150	3.777	42.61	27.49	19.00	66.81	14.19			43.00	28.81
19.00			8.800 10.323	4.207	38.77	26.49 24.49	23.00	68.81 70.81	15.11	11.560			26.81 24.81
[25.00]	68.49	13.73	12.107	4.510	36.22	22.49	25.00	72.81	12.59	16.293	6.213		22.81
25.00 27.00	70.49	13.19	14.243	4.813	33.68	20.49	27.00	74.81 76.81 78.81	12.08	19.480	6.747	32.89	20.81
27.00	72.49	12.56	16.843	5.110	31.15	18.49	29.00	76.81	111.56	23.367		30.37	18.81 16.81
29.00	76.49	11.64	23.720	5.617	26.13	14.49	33.00	80.81	10.58	34.00	8.30	25.39	14.81
33.00	78.49	11.14	28.353	5.780	23.63	12.40	35.00	82.81	10.12	40.97	8.73	22.93	12.81
35.00				5.82	21.14	10.49	37.00	84.91	9.63		9.03	20.44	10.81
37.00 39.00		9.69		5.70 5.32	18.67 16.18	8.49 6.49	41.00	86.81	9.16 8.70	62.22 76.65	9.10 8.78	17.97 15.51	8.81 6.81
41.00	86.49	9.23	61.43	4.51	13.72	4.49		90.81	8.25	98.17	3.87	13.06	4.81
43.00	88.49	8.77	76.05	3.07	11.26	2.49	45.00	92.81	7.81	125.13	2.88	10.62	2.81
45.00		8.32 8.21	95.25 100.01	.67	8.81 8.21	.49	47.00	94.81 95.62	7.36	164.43 179.47	.90	8.17 7.18	.81
20.23	30.30	0.21	100.01		0.21		*1.01	50.0Z	(*18	718.41	<u> </u>	(,,18	



TABLE 1. - DETAILED NOZZLE DESIGN PARAMETERS - Concluded  $[\gamma = 1.400]$ 

1	2	3	1 4	5	-	7	<del>n ,     </del>	T .	-				-
			4		6	<del>-                                    </del>	1	2	3	4	5	6	7
Ψ \	¥ (4-)	β	$\frac{x}{A_t/2}$	$\frac{y}{A_t/2}$	\.\ <sup>\_</sup> .	٠,۵	Y-,	¥ .	β,	At/2	$\frac{y}{\Lambda_{t}/2}$	λ_	,α,
(deg)	(deg)	(deg)		<u> </u>	(deg)	(deg)	(deg)	(deg)	(deg)	,		(deg)	(deg)
	M <sub>f</sub> ,	9.00;	φ and Ψ <sub>f</sub>	, 49.6	6°					e and Y	·		
0	49.66	19.50	0	1.000		49.66	0.01		19.00	.690	1.000		51.16 51.15
.01	49.67	19.50	.673	1.387	69.15	49.65	.04	51.20	18.98	.900	1.560	70.10	51.12
.04 .07	49.70		.880		69.11	49.62 49.59	.07		18.97 18.96	1.070	1.620 1.667		51.09 51.06
.10	49.76	19.47	1.043	1.600	69.03	49.56	.13	51.29	18.95	1.123	1.700	69.98	51.03
	49.79 49.82	19.45 19.44	1.097		68.98 68.94	49.53 49.50	.16		18.94 18.93	1.173		69.94 69.90	51.00 50.97
	49.85		1.183	1.680	68.90	49.47	.25	51.41	18.91	1.273	1.793	69.82	50.91
.25		19.42	1.243		68.83	49.41 49.35	.31 .37		18.88 18.86	1.333			50.85 50.79
.37	50.03	19.37	1.353	1.780	68.66	49.29	.46	51.62	18.83	1.447	1.897	69.53	
.46 .55			1.410		68.54 68.42	49.20 49.11	.55 .64		18,80	1.507	1.937		50.61 50.52
.64	50.30	19.28	1.527	1.977	68.50	49.02	.73	51.89	18.73	1.617			
.73 .82			1.627	1.903	68.17	48.93 48.84	.92		18.71 18.68	1.667	2.033		
.91	50.57	19.19	1.667	1.957	67.94	48.75	1.00		18.65	1.757	2.090		
1.00			1.710		67.82 67.56	48.66	1.20		18.58	1.853	2.147	68.54	49.96
1.40			1.800 1.880		67.30	48.46 48.26	1.40		18.51 18.45	1.933	2.197		49.76 49.56
1.60 1.80			1.953		67.02	48.06	1.80	52.96	18.39	2.090	2.290	67.75	49.36
2.00			2.033		66.75 66.48	47.86 47.66	2.40		18.33 18.19	2.160	2.333 2.417	67.49	49.16 48.76
2.40	52.06	18.68	2.237	2.273	65.94	47.26	2.90	53.96	18.05	2.447	2.500	66.41	48.36
2.80 3.20		18.54 18.42	2.577	2.350 2.420	65.40	.46.86 46.46	3.20 3.60		17.92 17.79	2.580	2.582	65.88 65.35	47.96 47.56
3.60	53.26	18.29	2.637	2.487	64.35	46.06	4.00	55.16	17.67	2.863	2.740	64.83	47.16
4.50	53.66 54.16	17.99	2.773	2.560		45.66 45.16	4.50 5.00		17.50	3.023		64.16 63.51	46.66 46.16
5.00	54.66	17.82	3.087	2.720	62.48	44.66	5.50	56.66	17.19	3.370	3.020	62.85	45.66
5.50 6.00	55.16 55.66		3.250 3.440		61.83	44.16 43.66	6.50		17.03 16.87	3.567	3.127	62.19	45.16 44.66
6.50	56.16	17.35	3.610	2.983	60.51	43.16	7.00	58.16	16.71	3.943	3.323	60.87	44.16
	56.66 57.16		3.793 3.987	3.070		42.66 42.16	7.50 8.00		16.56 16.41	4.153	3.430 3.533	60.22 59.57	43.56 43.16
8.00	57.66	16.87	4.170	3.247		41.66	9.00	60.16		4.730	3.723	59.27	42.16
	58.66 59.66		4.530	3.410 3.590		40.66 39.66	10.00	61.16		5.180		56.97	
	60.66		5.400			38.66	12.00	63.16	15.51 15.21	5.683 6.213	4.190 4.433	55.67 54.37	40.16 39.16
	61.66 62.66		5.877	3.980		37.66	13.00	64.16		6.907		55.09	
	63.66		6.413 6.983	4.190		36.66 35.66	14.00	65.16		7.440 8.147	4.977 5.273	50.54	
15.00	64.66	14.79	7.613	4.643	49.45	34.66	16.00	67.16	14.10	8.890	5.573	49.26	35.16
	65.66		8.273 9.013	4.873 5.120		33.66 32.66	17.00	68.16		9.733		47.98	
18.00	67.66	13.96	9.800	5.377	45.62	31.66	19.00	70.16	13,28	11.613	6.583	45.44	32.16
	68.66 70.66		10.680 12.695	5.653 6.240		30.66 28.66	23.00	72.16 74.16		13.893		42.91	
23.00	72.66	12.62	15.183	6.887	39.28	26.66	25.00	76.16	11.72	20.190	9.103	37.38	26.16
	74.66		18,200 21,960	7.600 8.377	36.77 34.26	24.66 22.66	27.00	78.16 80.16	11,23	24.507	10,140	35.39 32.90	
29.00	78.66	11.10	26.533	9.190	31.76	20.66	31.00	82.16	10.26	36.67	12.57	30.42	20.16
33.00	80.66 82.66	10.61	32.20 39.47	10.96	29.27 26.81	18.66 16.66	33.00 35.00	84.18	9.77	45.37	13.97	27.93	
35.00	84.66	9.66	48.15	11.83	24.32	14.66	37.00	88.16			16.77	25.48 23.00	14.16
37.00	86.66	9.20 8.73			21.86	12.66 10.66	39.00 41.00	90.16	8.39	88.35	18.18	20.55	12.16
41.00	90.66	8.28	93.92	13.72	19.39 16.94	8.66	43.00			112.30 147.52	19.43 20.30	18.11 15.66	8.16
45.00 45.00	92.66	7.84	121.93	13.50	14.50	6.66	45.00	96.16	7.06	193.80	20.20	13.22	6.16
	96.66		157.95 211.03	9.00	12.05 9.61	4.66 2.66	47.00	98.16 100.16		262.5 356.2	18.1	10.79 8.36	4.16 2.16
49.00	98.66	6.52	250.0	5.5	7.18	.66	51.00	102.16	5.77	499.0	1.0	5.93	.16
49.66	20.68	5.38	306.7	0	6.38	0	DT • 16	102.32	0.74	509.7	0	5.74	0

TABLE II. - OVER-ALL NOZZLE DESIGN PARAMETERS  $\begin{bmatrix} \gamma = \ 1.400 \end{bmatrix}$ 

	<del></del>								
1	2	3	4	5	1	2	3	4	5
M	<pre>p and Yf* (deg)</pre>	Yf, Y (deg)	A <sub>f</sub> /A <sub>t</sub>	β <sub>f</sub> , β (deg)	Mf	φ and Ψf (deg)	Yf, Y (deg)	A <sub>f</sub> /A <sub>t</sub>	β <sub>f</sub> , β (deg)
1.00 1.02 1.04 1.06 1.08	.063 .175 .318 .484	0 •126 •351 •637 •968	1.0013	90.000 78.635 74.058 70.630 67.808	1.80 1.82 1.84 1.86 1.88	10.652	21.304	1.4390 1.4610 1.4836 1.5069 1.5307	33.749 33.329 32.921 32.523 32.135
1.10 1.12 1.14 1.16 1.18	.668 .867 1.080 1.304 1.537		1.0198	63.234 61.306	1.90 1.92 1.94 1.96 1.98	12.076 12.356 12.635		1.5804 1.6062	31.757 31.388 31.028 30.677 30. <b>535</b>
1.20 1.22 1.24 1.26 1.28	1.779 2.028 2.285 2.546 2.814		1.0504	55.052 53.751 52.528		14.011		1.7160 1.7451 1.7750	29.673
1.30 1.32 1.34 1.36 1.38	3.085 3.360 3.635 3.922 4.206	6.721 7.279 7.844		49.251	2.10 2.12 2.14 2.16 2.18	14.815 15.080 15.344		1.8690 1.9018 1.9354	28.145 27.859
1.40 1.42 1.44 1.46 1.48	4.493 4.782 5.073 5.365 5.663	9.565	1.1379	45.585 44.767 43.983 43.230 42.507	2.20 2.22 2.24 2.26 2.28	16.125	31.732 32.250 32.763 33.274 33.778	2.0409 2.0777 2.1154	26.262
1.50 1.52 1.54 1.56 1.58	6.248	11.906 12.495 13.085 13.675 14.270	1.1899 1.2042 1.2190	41.810 41.140 40.493 39.868 39.265	2.36	17.391 17.639	34.283 34.782 35.279 35.771 36.262	2.2333 2.2744 2.3164	25.533 25.300 25.070
1.60 1.62 1.64 1.66 1.68			1.2666 1.2835 1.3010	38.682 38.118 37.572 37.043 36.530		18.615 18.854	37.230		
1.70 1.72 1.74 1.76 1.78	8.905 9.198 9.490 9.783 10.073	18.397 18.981 19.566	1.3567 1.3764 1.3967	36.032 35.549 35.080 34.624 34.180	2.50 2.52 2.54 2.56 2.58	19.794 20.025	39.124 39.589 40.050 40.508 40.963	2.6864 2.7372	23.380

TABLE II. - OVER-ALL MOZZLE DESIGN PARAMETERS - Continued  $[\gamma = 1.400]$ 

1	2	3	4	5	1	2	3	4	5
Mf	φ and Ψ <sub>f</sub> + (deg)	Y <sub>f</sub> , Y (deg)	A <sub>f</sub> /A <sub>t</sub>	β <sub>f</sub> , β (deg)	Mf	φ and Ψ <sub>1</sub> (deg)	Y <sub>f</sub> , Y (deg)	A <sub>2</sub> /A <sub>t</sub>	β <sub>f</sub> , β (deg)
2.60 2.62 2.64 2.66 2.68	20.707 20.931 21.154 21.374 21.593	41.415 41.863 42.308 42.749 43.187	2.9511 3.0073 3.0647	22.620 22.438 22.259 22.082 21.909	3.30 3.32 3.34 3.36 3.38	27.611 27.782 27.952 28.120 28.288	55.222 55.564 55.904 56.241 56.576	5.6286 5.7358 5.8448 5.9558 6.0687	17.640 17.530 17.422 17.315 17.209
2.70 2.72 2.74 2.76 2.78	21.810 22.026 22.240 22.453 22.664	43.621 44.053 44.481 44.906 45.328	3.1830 3.2440 3.3061 3.3695 3.4342	21.738 21.571 21.405 21.243 21.082	3.40 3.42 3.44 3.46 3.48	28.619	56.908 57.238 57.564 57.888 58.210	6.1837 6.3007 6.4198 6.5409 6.6642	17.105 17.002 16.900 16.799 16.700
2.80 2.82 2.84 2.86 2.88	22.873 23.080 23.286 23.491 23.694		3.6359 3.7058	20.925 20.770 20.617 20.466 20.318	3.50 3.52 3.54 3.56 3.58	29.973 29.581	58.530 58.847 59.162 59.474 59.784	6.7896 6.9172 7.0470 7.1791 7.3135	16.602 16.504 16.409 16.314 16.220
2.90 2.92 2.94 2.96 2.98	24.490	47.790 48.190 48.586 48.980 49.370	3.8498 3.9238 3.9993 4.0763 4.1547	20.027 19.885 19.745	3.60 3.62 3.64 3.66 3.68	30.198 30.350 30.500	60.091 60.397 60.700 61.000 61.299	7.4501 7.5891 7.7304 7.8742 8,0204	16.128 16.036 15.946 15.856 15.768
3.00 3.02 3.04 3.06 3.08	25.071 25.261	50.523	4.2346 4.3160 4.3989 4.4835 4.5696		3.70 3.72 3.74 3.76 3.78	30.747 30.944 31.090 31.235 31.379	61.595 61.889 62.181 62.471 62.758	8.1690 8.3202 8.4759 8.6302 8.7891	15.680 15.594 15.508 15.424 15.340
3.10 3.12 3.14 3.16 3.18	25.825 26.010 26.193 26.375	51.650 52.020	4.6573 4.7467 4.8377 4.9304	18.819 18.694 18.570 18.449 18.328	3.80 3.82 3.84 3.86 3.88	31.663 31.804 31.943	63.044 63.327 63.608 63.887 64.164	8.9506 9.1148 9.2817 9.4513 9.6237	15.258 15.176 15.095 15.015 14.936
3.20 3.22 3.24 3.26 3.28	26.735 26.913 27.089 27.265	53.470 53.826 54.179 54.530	5.1210 5.2189 5.3186 5.4201	18.210 18.093 17.977 17.863 17.751	3.90 3.92 3.94 3.96 3.98	32.220 32.356	64.440 64.713 64.984 65.253 65.520		14.857 14.780 14.703 14.627

TABLE II. - OVER-ALL NOZZLE DESIGN PARAMETERS - Continued  $[\gamma = 1.400]$ 

1	2	3	4	5	1	2	3	4	5
ur	φ and Ψf <sup>+</sup> (deg)	Yf, Y (deg)	A <sub>f</sub> /A <sub>t</sub>	β <sub>f</sub> , β (deg)	uf	<pre>p and f (deg)</pre>	Yr, Y (deg)	A <sub>f</sub> /A <sub>t</sub>	β <sub>f</sub> , β (deg)
4.00 4.05 4.10 4.25 4.30 4.35 4.40 4.55 4.60 4.65 4.70 4.75	32.892	65.785 66.439 67.085 67.714 68.334 68.945 69.541 70.128 70.707 71.274 71.833 72.380 72.919 73.448 73.969 74.483	10.719 11.207 11.715 12.243 12.791 13.363 13.955 14.571 15.210 15.874 16.562 17.277 18.018 18.787 19.583 20.409	14.478 14.295 14.117 13.943 13.774 13.609 13.448 13.290 13.137 12.986 12.840 12.696 12.556 12.419 12.284	5.50 5.55 5.60 5.65 5.70 5.85 5.90 5.95 6.00 6.10 6.20 6.25	40.622 40.821 41.016 41.209 41.397 41.585 41.768 41.950 42.128 42.303 42.477 42.649	81.244 81.643 82.032 82.418 82.795 83.171 83.537 83.900 84.257 84.607 84.955 85.299 85.634 85.968 86.296 86.618	36.869 38.281 39.741 41.246 42.796 44.400 46.050 47.754 49.507 51.318 53.178 55.101 57.077 59.114 61.210 63.370	
4.75 4.80 4.85 4.90 4.95 5.00 5.15 5.20 5.25 5.30 5.35 5.40 5.45	37.241 37.493 37.741 37.985 38.225 38.460 38.691 38.920 39.146 39.367 39.585 39.799 40.008 40.216 40.422	74.986 75.483 75.970 76.451 76.921 77.383 77.841 78.293	20.409 21.263 22.151 23.067 24.018 25.000 26.018 27.069 28.159 29.283 30.446 31.649 32.893 34.174 35.501	12.025 11.899 11.776 11.655 11.537 11.421 11.308 11.197 11.088	6.25 6.30 6.35 6.40 6.55 6.60 6.65 6.70 6.75 6.80 6.85 6.90 6.95	43.469 43.625 43.780 43.934 44.084 44.233	86.938 87.251 87.561 87.868 88.169 88.466 88.759 89.051 89.336 89.618 89.895 90.170 90.442 90.710	65.589 67.877 70.228 72.647 75.134 77.695	9.133 9.061 8.989 8.919 8.850 8.782 8.715 8.649 8.584 8.520 8.457 8.334 8.333 8.273

TABLE II. - OVER-ALL NOZZLE DESIGN PARAMETERS - Concluded  $[\gamma = 1.400]$ 

1	2	3	4	5	1	2	3	4	5
Mf	φ and Ψ † (deg)	Yf, Y (deg)	A <sub>f</sub> /A <sub>t</sub>	β <sub>f</sub> , β (deg)	Mf	φ and Ψ <sub>f</sub> <sup>+</sup> (deg)	Yf, Y (deg)	A <sub>f</sub> /A <sub>t</sub>	β <sub>f</sub> , β (deg)
7.00 7.05 7.10 7.20 7.25 7.30 7.40 7.45 7.50 7.65 7.70 7.75 7.80 7.90 7.95 8.00 8.15 8.20	(deg) 45.487 45.618 45.746 45.873 45.999 46.122 46.245 46.365 46.485 46.603 46.720 46.835 46.949 47.061 47.172 47.283 47.391 47.499 47.604 47.708 47.813 47.916 48.117 48.215	90.974 91.237 91.492 91.746 91.999 92.244 92.491 92.731 93.206 93.441 93.671 93.898 94.345 94.345 94.345 94.567 94.783 94.998 95.417 95.627 95.832 96.033 96.234 96.431	104.143 107.492 110.931 114.459 118.080 121.794 125.605 129.513 133.520 137.629 141.842 146.159 150.585 155.120 159.770 164.527 169.403 174.418 179.511 184.744 190.109 195.597 201.215 206.964 212.846	8.213 8.155 8.097 8.040 7.928 7.873 7.820 7.766 7.714 7.662 7.611 7.561 7.561 7.414 7.462 7.414 7.366 7.319 7.272 7.226 7.181 7.136 7.092 7.048 7.005	8.50 8.65 8.60 8.65 8.75 8.80 8.85 8.90 9.05 9.05 9.10 9.25 9.30 9.35 9.40 9.45 9.50 9.60 9.65 9.70	(deg) 48.786 48.878 48.969 49.059 49.147 49.234 49.321 49.407 49.576 49.660 49.741 49.823 49.904 49.983 50.063 50.141 50.219 50.295 50.371 50.445 50.520 50.594 50.738	97.575 97.757 97.938 98.118 98.294 98.643 98.643 99.153 99.320 99.483 99.4647 99.808 99.967 100.282 100.438 100.438 100.591 100.742 100.891 101.188 101.334 101.476	251.086 257.974 265.014 272.211 279.567 287.084 294.766 302.615 310.633 318.823 327.190 335.733 344.458 363.368 362.463 371.749 381.228 390.902 400.775 410.851 421.131 431.620 442.322 453.236 464.370	6.756 6.717 6.677 6.639 6.600 6.562 6.525 6.488 6.451 6.379 6.344 6.379 6.240 6.240 6.240 6.173 6.140 6.107 6.074 6.074 6.074 6.074 6.074 6.074 6.074
8.25 8.30 8.35 8.40 8.45	48.312 48.410 48.506 48.599 48.694		218.865 225.022 231.320 237.763 244.350	6.962 6.920 6.878 6.837 6.796	9.75 9.80 9.85 9.90 9.95	50.811 50.882 50.951 51.021 51.090	101.623 101.764 101.903 102.042 102.180		5.887 5.857 5.827 5.797 5.768 5.739

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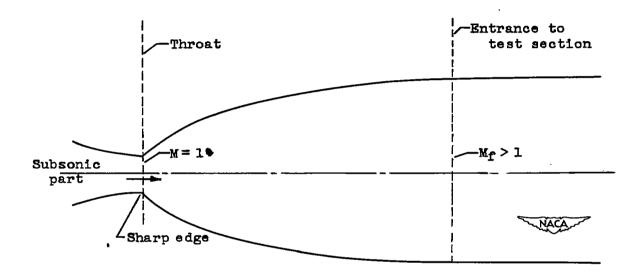
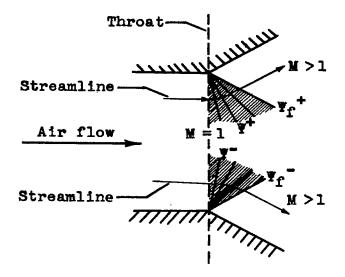
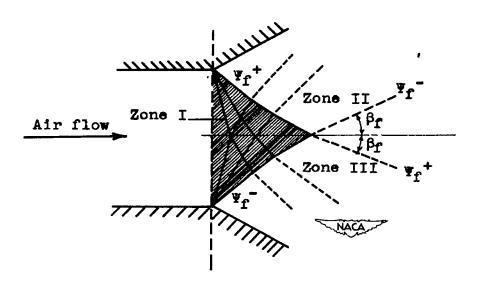


Figure 1. - Sharp-edge-throat supersonic nozzle of minimum length.



(a) Expansion waves represented by & finite number of characteristics.

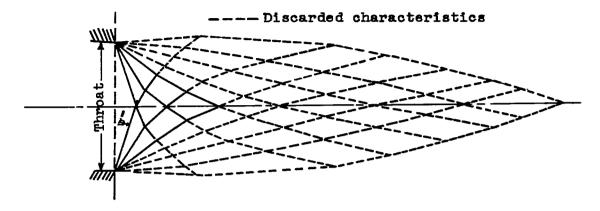


(b) Wave pattern formed by interaction of two expansion waves.

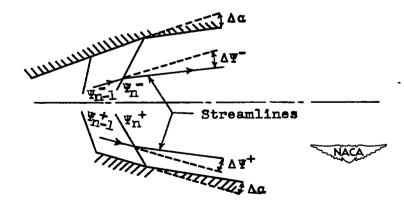
Figure 2. - Schematic representation of expansion waves by characteristics.

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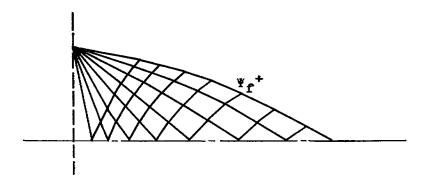


(c) Kernel formed from kernel corresponding to higher Mach number.

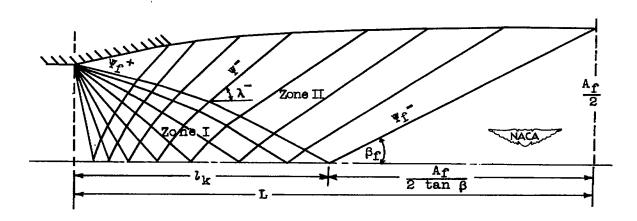


(d) Suppression of expansion wave by bending wall.

Figure 2. - Concluded. Schematic representation of expansion waves by characteristics.

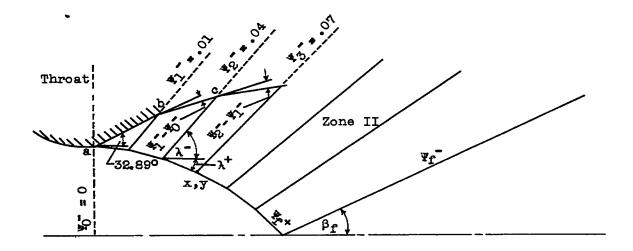


(a) Kernel.

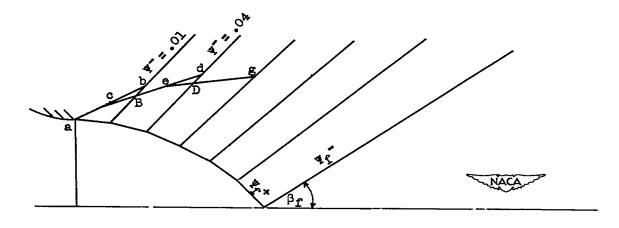


(b) Wave pattern and wall contour.

Figure 3. - Complete wave pattern and wall contour of graphically designed nozzle with sharp-edge throat.



(a) Conventional development.



(b) Averaging development.

Figure 4. - Development of wall contour from bounding  $\Psi_{\Gamma}^+$  characteristic of kernel.

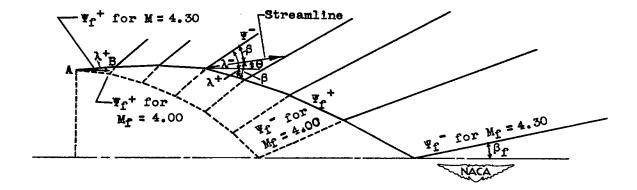


Figure 5. - Method of determining bounding characteristic  $\Psi_{\Gamma}^+$  for a desired Mach number from a known adjacent characteristic.

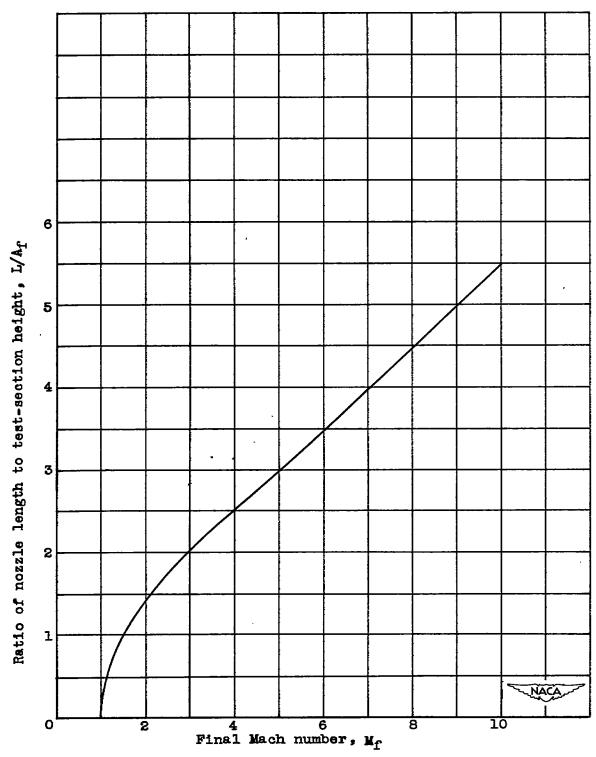


Figure 6. - Length of sharp-edge-throat nozzles.